ASSESSMENT OF POSITION ERROR IN DUAL-FREQUENCY IRNSS RECEIVERS: INSIGHTS FROM THE BANGALORE REGION

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Abstract
In satellite based navigation systems (such as GPS, the Global Positioning System or IRNSS, the Indian Regional Navigation Satellite System), unlike a single-frequency user-receiver, the dual-frequency user-receiver doesn't require the appropriate ionospheric models for correcting the range error due to ionospheric propagation. Instead, in such dual-frequency receiver, the ionospheric range error of the observed satellite is computed in terms of the difference between the measured pseudo-ranges at two operating carriers of the satellite. Consequently, in such receiver, the ionospheric range error estimation will be more accurate, because the contribution to the range error due to all the frequency independent error-sources will be nullified while taking the difference between the two measured pseudo-ranges. However, in such dual-frequency receiver, the ionospheric range error estimation will be perturbed by the presence of several frequency dependent error-sources. In this paper an algorithm has been formulated to estimate the position error of a dual-frequency IRNSS user-receiver over the Bangalore region.

Keywords: Dilution of Precision (DOP), Global Positioning System (GPS), Indian Regional Navigation Satellite System (IRNSS), Root Mean Square (RMS), Root Sum Square (RSS),Total Electron Content (TEC), User Equivalent Range Error (UERE).

1. Introduction
Like the GPS (Global Positioning System) receiver, the final navigation solution (in terms of position coordinates, x, y, z and timing information, t) in a single or dual-frequency IRNSS ((Indian Regional Navigation Satellite System) user-receiver is obtained in two steps. In the first step, the receiver measures the pseudoranges simultaneously for a minimum of four satellites to yield the basic set of four range equations. In the next step, the navigation processor within the receiver solves the basic set of four range equations to get the user's position and timing (x, y, z and t). Since the range equations are second-order nonlinear equations, typically the solution of these equations is performed using multi-dimensional Newton-Raphson method and a reasonable guess of initial receiver position. This solution is, of course, an approximation, so an iterative approach is required whereupon the most recent solution becomes the initial guess and the solution process is repeated until the desired accuracy (or the desired position error) is obtained [1-5]. Hence, for the user-receiver, it is important to have the knowledge about the minimum position error available or the best position accuracy achievable over the particular region within the service (or, coverage) area.

In global or regional navigation satellite systems, the group delay of the PRN (Pseudo Random Noise) code or the phase advance of the carrier wave of the complex navigation signal caused by the ionosphere constitutes almost 50 to 70 percent [6-8] of the total range error. In a single-frequency user-receiver, the range error correction due to the ionosphere is done by the various mathematical models of the ionosphere available in the literature [9-12], namely the ionospheric grid mode, the Klobuchar model etc. All these models require the TEC (Total Electron Content) mapping of the ionosphere over the service area. Due to the complex nature of the ionosphere, the TEC map over the region will vary in a complex manner and consequently these mathematical models cannot correct the ionospheric range error completely. Even using the best mathematical model, a single-frequency user- receiver can correct the ionosphere range error up to 60 % of the total range error due to ionospheric effects. The remaining 40% of the ionospheric range error will remain uncorrected and will be treated as the modelling error, which in turn contributes significantly to the position error of the single-frequency user.

In dual-frequency receivers no modelling of the ionosphere is required because the availability of two signals which have undergone the same ionospheric effects is exploited. In the absence of measurement errors, the ionospheric group delay or phase advance is estimated and fully mitigated via the difference between two available pseudo-range measurements at both the carriers [13]. Consequently, the dual-frequency receiver will find its position more accurately than that of its single-frequency counterpart.

2. Position Error Estimation in a Dual-Frequency User-Receiver

In a dual-frequency IRNSS user-receiver, the position error estimation over a certain region in India could be measured analytically from simultaneous pseudorange measurement over a short period at both the carriers from an IRIMS (IRNSS Range and Integrity Monitoring Station) located in that region by a dual frequency IRNSS reference receiver. On 30th April, 2018, the pseudo-range measurements of all the seven operating IRNSS satellites (namely, IRNSS-1B, 1C,1D,1E,1F, 1G and 1I) at both the L5 band and S band carriers had been carried out simultaneously using a multi-channel dual-frequency IRNSS reference-receiver from an IRIMS (IRNSS Range and Integrity Monitoring Station) earth station located in Bangalore [14].

2.1 The Basic Principle

The position error in a dual-frequency user-receiver mainly depends on two main factors as stated below:

a) The contribution to the range error due to frequency- dependent error sources. The example of such error-sources are the Doppler effect, the multipath propagation, the antenna phase centre variations, the receiver's figure of merit etc. These error-sources are broadly categorized into two types- (i) the fluctuating error sources, whose properties fluctuate randomly over the time and (ii) the constant error sources, whose properties do not change over the time.

b) The geometric error of the satellites in the constellation.

The variations in the properties of the **fluctuating error sources** with time will cause a random fluctuation in the ionospheric range error estimation. In general, the ionospheric property such as the TEC (Total Electron Content) values within the ionosphere does not vary [6-8] over the short period. Hence, by measuring the random fluctuations of the ionospheric range error values over a short period will enable us to estimate the position error in a dual frequency user-receiver due to the random property variations of such error sources.

On the other hand, the position error due to the constant offset caused by the frequency dependent **constant error sources** could be estimated by measuring the peak to peak variations of the mean ionospheric range errors across all the satellites.

Also the position error due to the error in satellite geometry (i.e., the constellation error) is taken into account by the appropriate DOP (Dilution of Precession) factors. The DOP has different forms, namely, HDOP (Horizontal Dilution of Precession), VDOP (Vertical Dilution of Precession), PDOP (Position Dilution of Precession) and TDOP (Time Dilution of Precession) etc.

In fact, from the range error variations of these satellites one could calculate the RMS (Root Mean Square) errors or the Standard Deviations in ionospheric range error estimates for all the satellites by statistical approach. The UERE (User Equivalent Range Error) **due to fluctuating error sources** could then be computed at both the carriers as the arithmetic mean of all these RMS (Root Mean Square) errors calculated at L5 and S band carriers. On the other hand, the UERE **due to constant error sources** is calculated as the peak to peak variations in the mean ionospheric range errors computed across the satellites. The overall UERE is then calculated as the RSS (Root Sum Square) value of both these UERE values.

Consequently, the position error (3-sigma value) will be obtained by multiplying the overall UERE by 3 and the appropriate DOP factors [15]

2.2 The Position Error Estimation Algorithm

The algorithm for the position error estimation could be expressed as follows:

Step 1:

From the pseudo-range measurement data at L5 and S band RF carriers, compute the ionospheric range errors, dR_{L5} and dR_S at both the RF carriers as follows [1]:

For L5 band carrier:

$$dR_{L5} = 1.286[R\ (L5) - R\ (S)] \quad (1)$$

For S band carrier:

$$dR_S = 0.286[R\ (L5) - R\ (S)] \quad (2)$$

Where, R (L5) = measured pseudorange at L5 band carrier
R (S) = measured pseudorange at S band carrier

Step 2:
From the ionospheric range error variations over short durations (taking at least 1000 data points or samples), compute the RMS errors / Standard Deviations, σ_{L5}, σ_S at both the RF carries for all the seven satellites (n = 1 to 7) as follows:

For L5 band:
RMS error in ionospheric range error estimates for the n-th satellite,

$$(\sigma_{L5})_n = \left[\frac{\sum_{i=1}^{N}(x_i - x_o)^2}{N}\right]^{0.5} \quad (3)$$

For S band:
RMS error in ionospheric range error estimates for the n-th satellite,

$$(\sigma_S)_n = \left[\frac{\sum_{i=1}^{N}(y_i - y_o)^2}{N}\right]^{0.5} \quad (4)$$

Here, N = Number of data points or samples (Usually, N ≥1000)

x_i = i-th sample of the ionospheric range error estimate at L5 band = $(dR_{L5})_i$
y_i = i-th sample of the ionospheric range error estimate at S band = $(dR_S)_i$

x_o and y_o are the mean values of the ionospheric range error estimates at L5 band and S band respectively and are given by

$$x_o = \frac{\sum_{i=1}^{N}(dR_{L5})_i}{N} \quad (5)$$

$$y_0 = \frac{\sum_{i=1}^{N}(dR_S)_i}{N} \quad (6)$$

Step 3:
Compute the User Equivalent Range Errors, $UERE_{L5}$ and $UERE_S$ by taking the arithmetic mean of all the RMS errors that have been calculated at L5 and S band carriers respectively (as shown in Step 2) corresponding to all the seven satellites. To compute use the following equations:

UERE at L5 band:

$$UERE_{L5} = \frac{\sum_{n=1}^{7}(\sigma_{L5})_n}{7} \quad (7)$$

UERE at S band:

$$UERE_S = \frac{\sum_{n=1}^{7}(\sigma_S)_n}{7} \quad (8)$$

Step 4:
Compute the User Equivalent Range Error due to the frequency-dependent fluctuating error sources ($UERE_f$,) as the arithmetic mean of the above two UEREs. That is

$$UERE_f = \frac{UERE_{L5} + UERE_S}{2} \quad (9)$$

Step 5:
From the ionospheric range errors estimates (as computed in Step 1) find their mean values at both the L5 and S band carriers for all the seven satellites using the equations 5 and 6 as shown in Step 2.

Next find the peak to peak variations in the mean ionospheric range errors at both L5 and S band carriers across all the seven satellites. Finally, calculate the User Equivalent Range Error due to the frequency-dependent constant error sources ($UERE_c$) as follows:

$$UERE_c = \frac{\frac{\Delta_{L5}}{2} + \frac{\Delta_S}{2}}{2} \quad (10)$$

Where,
Δ_{L5} = Peak to peak variation in the mean ionospheric range errors at L5 band across all the seven satellites
Δ_S = Peak to peak variation in the mean ionospheric range errors at S band across all the seven satellites

Step 6:
Compute the overall User Equivalent Range Error ($UERE_{over}$) as the Root Sum Square (RSS) value of the above two UEREs as calculated in Steps 4 and 5 above. That means,

$UERE_{over} = [(UERE_f)^2 + (UERE_c)^2]^{0.5}$ (11)

Step 7:
Compute the overall position error (1, 2 and 3-sigma values) in horizontal plane (i.e., in two dimensions), ΔP_{over} in a dual-frequency receiver as follows:

1-Sigma Error:
$\Delta P_{over} = UERE_{over} \times 2.2$ (12a)

2-Sigma Error:
$\Delta P_{over} = 2 \times UERE_{over} \times 2.2$ (12b)

3-Sigma Error:
$\Delta P_{over} = 3 \times UERE_{over} \times 2.2$ (12c)

(Taking the HDOP factor over Indian mainland ≈ 2.2 as shown in Figure 3 below)

Step 8:
Compute the overall position errors (1, 2 and 3-sigma values) in vertical plane (i.e., in two dimensions), ΔP_{over} in a dual-frequency receiver as follows:

1-Sigma Error:
$\Delta P_{over} = UERE_{over} \times 2.5$ (13a)

2-Sigma Error:
$\Delta P_{over} = 2 \times UERE_{over} \times 2.5$ (13b)

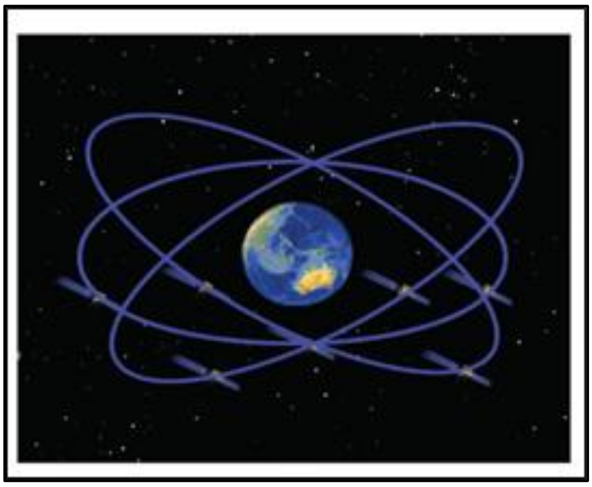
3-Sigma Error:
$\Delta P_{over} = 3 \times UERE_{over} \times 2.5$ (13c)

(Taking the VDOP factor over Indian mainland ≈ 2.5 as shown in Figure 3 below)

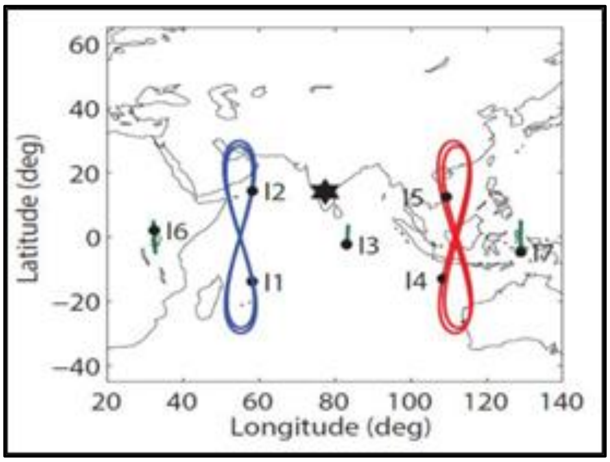
3. Pseudo-range Measurements of the IRNSS Satellites from Bangalore Earth Station

The space segment of the IRNSS system is the constellation of seven operating satellites in the sky. Out of these seven satellites, three are geostationary orbit satellites and lie in the equatorial plane. The remaining four are the geosynchronous orbit satellites and lie in two inclined planes with two satellites per plane, the inclination of each plane being 29 degrees.

The 7 Satellites IRNSS constellation and their ground tracks are shown in Figures 1(a) and 1 (b). The seven operating IRNSS satellites and their orbit details are shown [16-17] in the Table 1 below.



(a)



(b)

Figure 1: (a) The 7 Satellites IRNSS Constellation and (b) The ground tracks of IRNSS Satellites

Table 1: The seven operating IRNSS satellites and their orbit details

Satellites	**Orbit Types**	**Orbital Altitudes (Kms)**	**Orbital Slots (Latitudes)**
IRNSS-1B	Geosynchronous	36,000	55^0East
IRNSS-1C	Geostationary	36,000	83^0East
IRNSS-1D	Geosynchronous	36,000	111.75^0 East
IRNSS-1E	Geosynchronous	36,000	111.75^0East
IRNSS-1F	Geostationary	36,000	32.5^0East
IRNSS-1G	Geostationary	36,000	129.5^0East
IRNSS-1I	Geosynchronous	36,000	55^0 East

The IRNSS system is expected to provide the position accuracy of better than 20 m in the primary service area. The coverage area and the HDOP and VDOP contour plots are shown [17-19] in Figures 2 and 3 below.

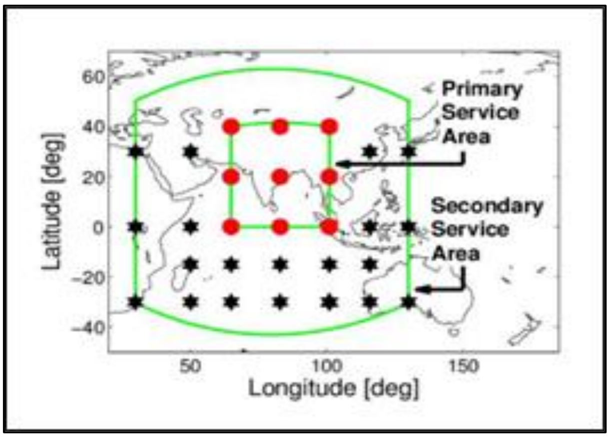
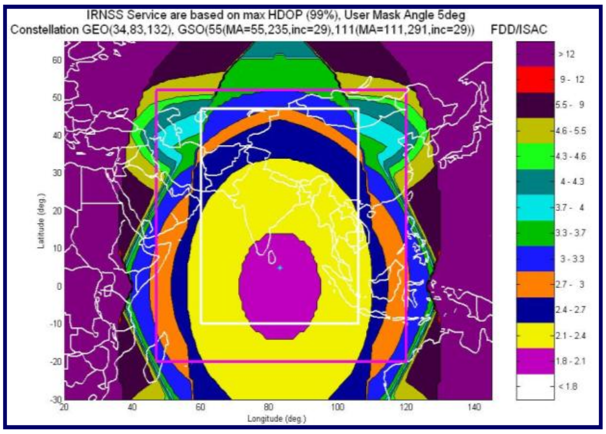
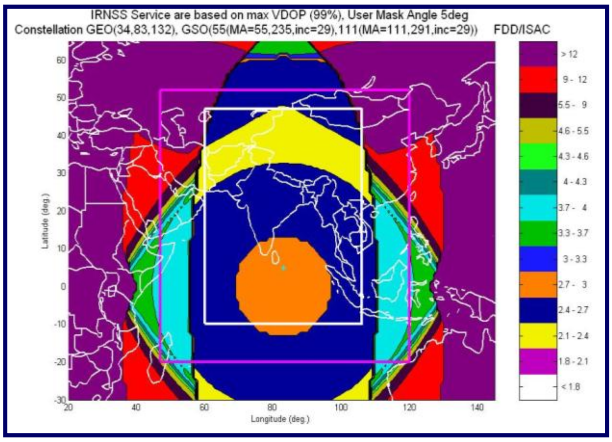


Figure 2: Primary (solid dots) and Secondary (stars) service area locations



(a)



(b)

Figure 3: 7 Day-averaged IRNSS (a) HDOP (Horizontal Dilution of Precision) and (b) VDOP (Vertical Dilution of Precision) contour maps.

Using the dual-frequency IRNSS reference -receiver (the receiver whose clock is synchronized with the IRNSS network time), the pseudo-ranges of all the acting IRNSS satellites in the constellation (as shown in Table 1) were measured at both the L5 and S band carriers (at every second intervals) from IRIMS earth station, Bangalore on 30th April, 2018 from about 14:20 hours. These measured pseudo-range data (for a short duration of about 17 minutes) for all the seven IRNSS satellites at L5 and S band carriers are plotted respectively in Figures 4 and 5 below:

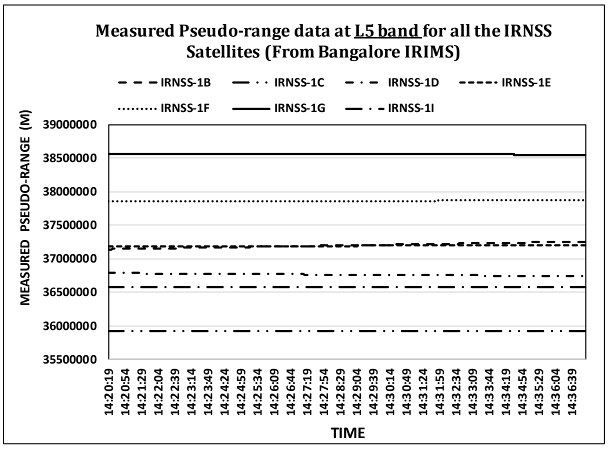


Figure 4: Measured Pseudo-range data at L5 band for all the IRNSS Satellites from Bangalore IRIMS

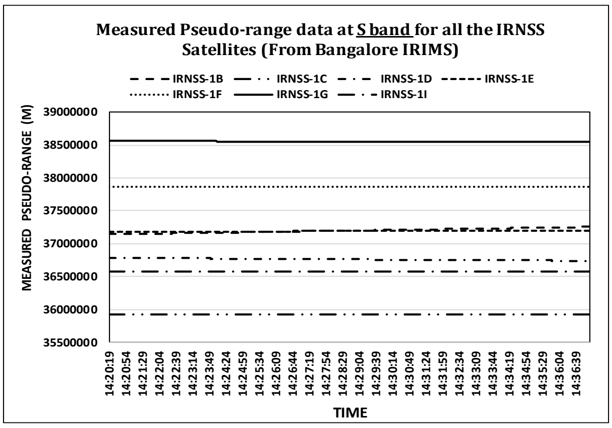


Figure 5: Measured Pseudo-range data at S band for all the IRNSS Satellites from Bangalore IRIMS

3.1 Estimation of Ionospheric Range Errors over Bangalore Region

Using these pseudo-range data (as shown in Figures 4 and 5 above), the ionospheric range errors at both the carriers were computed using the equations 1 and 2(as shown in step 1 in section 2.2) over a short duration (about 17 minutes) of time.

The ionospheric range error variations as computed over this short interval of time for all these IRNSS satellites are plotted in Figures 6 to 12 below.

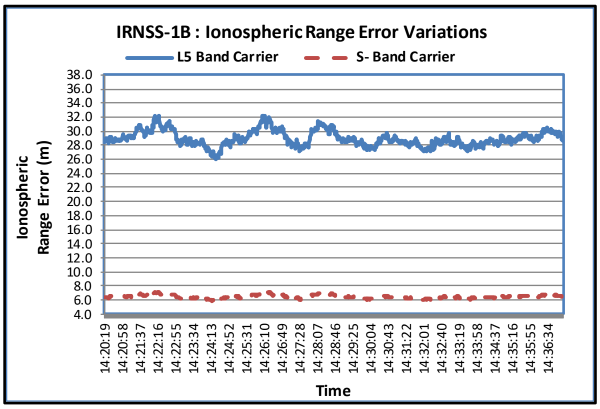


Figure 6: Ionospheric range error variation over short period of time for IRNSS-1B satellite

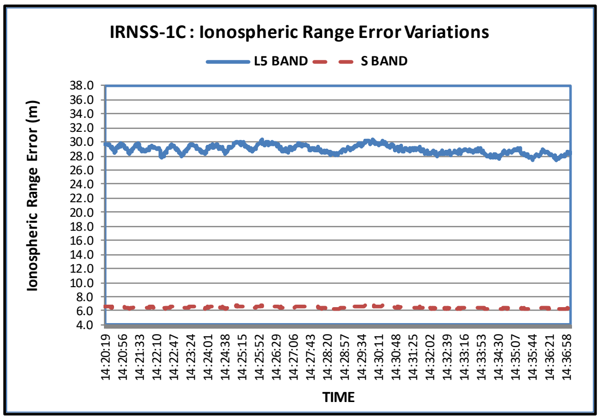


Figure 7: Ionospheric range error variation over short period of time for IRNSS-1C satellite

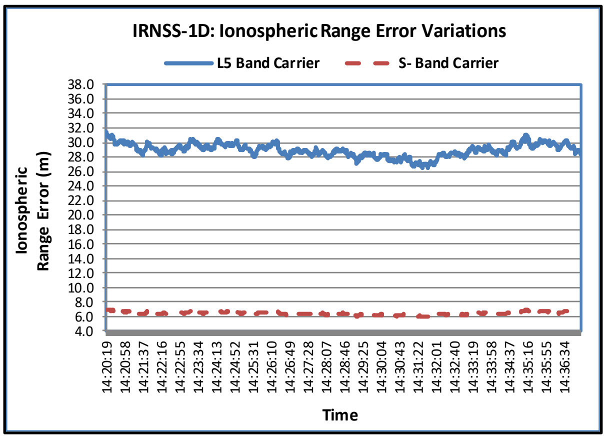


Figure 8: Ionospheric range error variation over short period of time for IRNSS-1D satellite

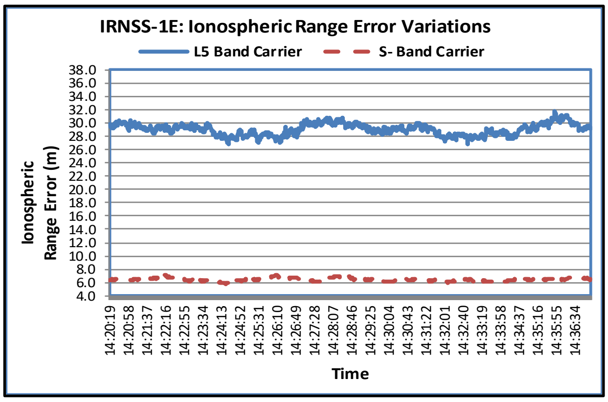


Figure 9: Ionospheric range error variation over short period of time for IRNSS-1E satellite

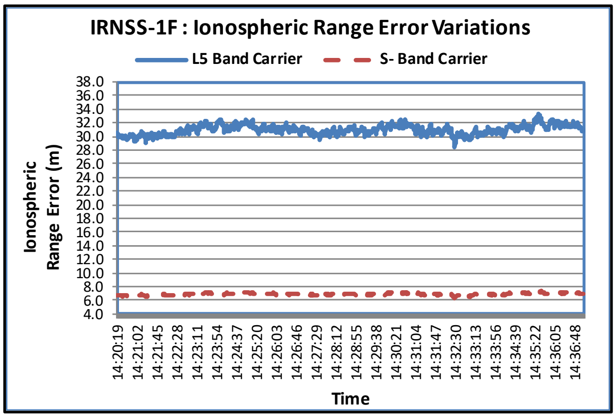


Figure 10: Ionospheric range error variation over short period of time for IRNSS-1F satellite

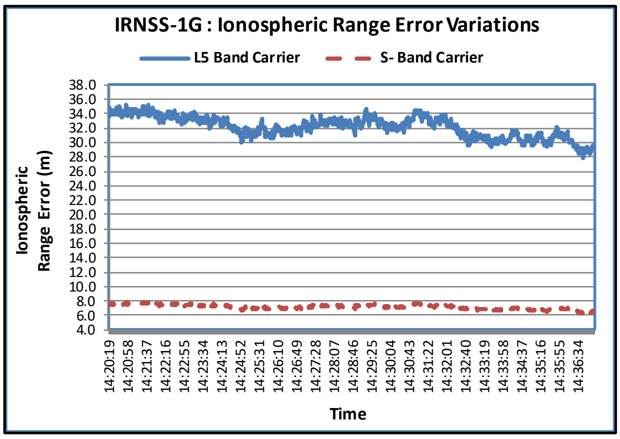


Figure 11: Ionospheric range error variation over short period of time for IRNSS-1G satellite

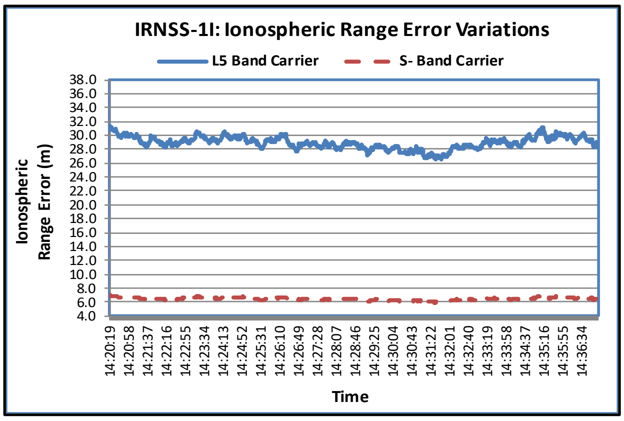


Figure 12: Ionospheric range error variation over short period of time for IRNSS-1I satellite

From the ionospheric range error data (as shown in Figures 6 to 12) the RMS Errors/Standard Deviations in the ionospheric range error estimates at both the carriers have been computed (with number of samples, N = 1012) using equations 3 and 4. Also, the mean values of the ionospheric range errors at both the carriers for all the seven satellites have been computed using the equations 5 and 6. The computed RMS errors in the ionospheric range error estimates and the mean values of ionospheric range errors for all the seven IRNSS satellites are shown in the Table 2 below. The values computed are also plotted in Figures 13and 14 below.

Table 2: The RMS errors and the mean value of the ionospheric range estimates for all the seven satellites

Satellites	**RMS Errors (standard deviations) in meter at**		**Mean value of Ionospheric Range Error in meter at**	
	L5 Band	S Band	L5 Band	S Band
IRNSS-1B	1.10	0.25	28.9	6.4
IRNSS-1C	0.60	0.14	28.9	6.4
IRNSS-1D	0.90	0.20	28.9	6.4
IRNSS-1E	0.87	0.20	29.1	6.5
IRNSS-1F	0.87	0.20	29.1	6.5
IRNSS-1G	1.48	0.33	32.1	7.1
IRNSS-1I	0.84	0.20	27.9	6.2

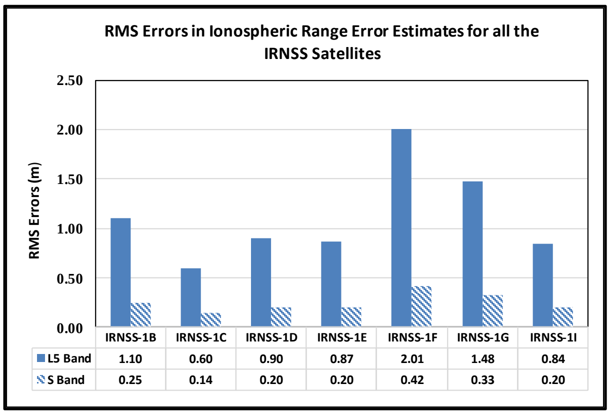


Figure 13: RMS Errors in Ionospheric Range Error Estimates for all the IRNSS Satellites

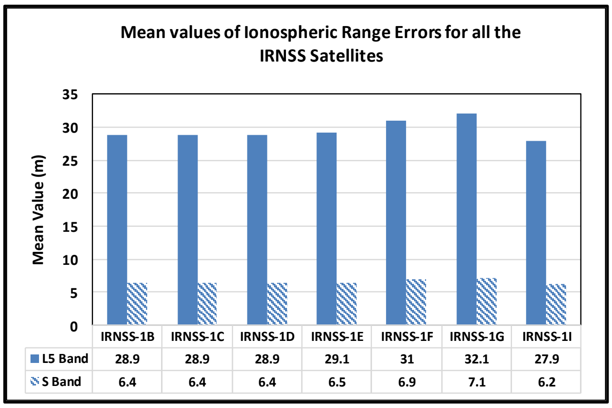


Figure 14: Mean values of Ionospheric Range Errors for all the IRNSS Satellites

3.2 Estimation of the Position Errors over Bangalore Region

From the ionospheric range error fluctuations of all the IRNSS satellites (as shown in Figures 6 to 12), the RMS errors in ionospheric range error estimates for all these satellites are computed using the equations 3 and 4 at L5 and S band carriers respectively (as indicated in section 2.2). Consequently, the UEREs for both the fluctuating and constant error-sources at both the carriers are computed using the equations 9 and 10. Finally, the overall UERE and the final position errors over Bangalore region are estimated using the equations 11, 12 and 13 in Steps 6,7 and 8 (as indicated in section 2.1). The computed results are shown in the Tables 3 and 4 below.

Table 3: Estimated User Equivalent Range Errors (UERE)

Sources of Errors	UERE Symbol	Estimated UERE Value(m)
Frequency-dependent Fluctuating Error sources	$UERE_f$	0.68
Frequency-dependent Constant Error sources	$UERE_c$	1.28
Combined Effect of both the above sources	$UERE_{over}$	1.45

Table 4: Estimated Position Errors (m) over Bangalore Region

Error Type	Estimated Position Error(m) in Horizontal plane	Estimated Position Error (m) in Vertical plane
1 –Sigma error	3.2	3.6
2-Sigma error	6.4	7.2

3-Sigma error	9.6	**10.8**

The estimated position errors (as shown in the Table 4 above) over Bangalore region in a dual-frequency IRNSS user-receiver are plotted in the Figure 15 below:

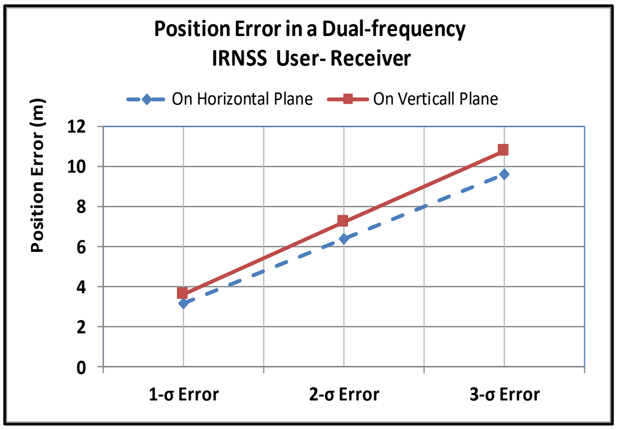


Figure 15: Estimated position errors over Bangalore region in a dual-frequency IRNSS user-receiver

Conclusion

From the above analytical computations, it has been observed that the position error (3-Sigma value) in a dual-frequency IRNSS use-receiver is about 10 meters (in both horizontal and vertical directions) over Bangalore region. This was confirmed by measuring the position of a dual frequency IRNSS reference receiver from its pre-defined location in Bangalore.

As part of the ground segment of IRNSS system, there are sixteen IRIMS earth stations spread across the country. In the similar way as done from IRIMS in Bangalore, the pseudo-ranges of all the IRNSS satellites could be measured simultaneously from all the remaining IRIMS within the country and thereby the position errors for different parts of the country could be estimated. The average of all the position errors could then be declared as the existing position error over the Indian mainland. In practice, the existing position error that is estimated in this way could be updated at regular interval of time by continuously measuring the pseudo-ranges over the day and night throughout the year. These errors could then be up linked at regular intervals to all the satellites in the constellation for their broadcast in the navigation message. The IRNSS user receiver (either single or dual frequency) upon receiving the navigation message (Almanac dada) will have the knowledge about the existing position error over the Indian mainland and thereby will compute its position more accurately in convenient and time-efficient manner.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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